

Syllabus: Theory Functions of a Real Variable Preliminary Examination

BACKGROUND: Students should know a little TOPOLOGY (such as open and compact sets, Hausdorff spaces, Urysohn's lemma, etc.) and METRIC SPACES (Metrics, metric topology, continuity, the Cauchy condition and completeness, etc).

MEASURES: Sigma-algebras, Borel sets, outer measures, monotone class theorem, Borel measures, regularity properties, Lebesgue measure on \mathbb{R}^n , signed and complex measures, Jordan decomposition, total variation of a measure, absolute continuity, product measures.

INTEGRATION: Measurable functions, simple functions, approximation properties, integration of non-negative functions, integration of complex and real valued functions, convergence theorems for integrals (e.g., Fatou, monotone, dominated convergence), almost everywhere convergence, Lusin's theorem, Egoroff's theorem, the Fubini-Tonelli theorem).

DIFFERENTIATION: Bounded variation, absolute continuity, Lebesgue decomposition and Radon-Nikodym theorem, fundamental theorem of calculus, differentiation and integration of measures and functions on \mathbb{R}^n , and basic connections with probability theory (distributions, density, independence).

BASICS of FUNCTIONAL ANALYSIS and L_p SPACES: Normed and Banach spaces, Hahn-Banach theorem, Hilbert spaces, Baire Category theorem, Open Mapping Theorem, Closed Graph Theorem, Principle of Uniform Boundedness, Applications to Fourier series, L_p spaces and their duals, completeness, convergence, density, $C(X)$, $C_c(X)$ and $C_0(X)$ and functionals on these: the Riesz representation/Markov-Kakutani theorems.

FOURIER TRANSFORM: Convolutions, Inversion, Plancherel's identity (if covered in course).

References:

- W. Rudin, Real and Complex Analysis, 3rd edition, McGraw-Hill, New York,
- G. B. Folland, Real Analysis, 2nd edition, Wiley Interscience, Somerset, NJ,
- H. L. Royden, Real Analysis, Prentice-Hall, Englewood Cl