# Incentives and Burnout: Dynamic Compensation Design With Effort Cost Spillover

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#### Abstract

Employee burnout has long plagued firms and salespeople are particularly susceptible. The prevalence of burnout indicates that work-related effort is not only costly in the present but has carryover effects into the future. The single-period principal-agent model commonly used to study sales force compensation design cannot fully account for this, as it effectively treats periods as independent. We incorporate 'effort cost spillovers' in a dynamic, two-period principal-agent model, with the salesperson's effort cost in the second period increasing in both her second-period *and* first-period efforts. We use this model to explore optimal compensation design and to consider the connection between incentives and burnout. If the firm and salesperson are forward-looking, we find that the firm can achieve its first-best outcome by committing to a contract for both periods in advance. Without commitment, the first-best remains achievable when effort spillovers are sufficiently small. Surprisingly, when the first-best is *not* achievable, the firm's equilibrium strategy may be to induce the salesperson to burn out in the first period (working so hard that she rejects any second-period contract that the firm would offer). This holds even when the salesperson cannot be replaced in the second period and the first-best outcome requires her to work in both periods.

Key words: sales force compensation, incentives, dynamic games, burnout, agency theory

## 1 Introduction

Since Basu et al. (1985), the dominant modeling paradigm for research on sales force incentive design has been agency theory. The vast majority of this research uses principal-agent models that represent a single period (e.g. Basu et al., 1985; Joseph and Thevaranjan, 1998; Godes, 2004), with the implicit assumption that a firm can follow single-period reasoning in each compensation period independently. Among the small subset that use multi-period or continuous-time models, the firm's and salesperson's utilities are assumed to be 'time-separable' (e.g., Lal and Srinivasan, 1993; Mantrala et al., 1997; Jerath and Long, 2020), with a few exceptions discussed below. In other words, the literature generally assumes that a salesperson's actions (e.g., her selling effort) at a particular time do not affect her utility or her ability to sell in the future.

However, there is good reason to believe that this is not the case in practice. Many people have experienced a situation in which they worked very hard for a period of time to meet a deadline or achieve a goal, then noticed a subsequent drop in their productivity or in their motivation to continue working. This could be due to fatigue or waning interest or to the desire to catch up on non-work-related activities like spending time with family and friends. Consider the following example, from Sullivan (2018): A few years ago, at her annual review, a friend was encouraged to pursue 'stretch' goals, and told that she could do more than she ever thought possible. Inspired, she worked hard to achieve these new goals, learning and growing along the way. However, she also sacrificed a lot by working long hours, as well as over weekends.

At the next review, her boss said: 'I knew you could do it! Let's bump up your goals again.' Still in the glow of success she agreed. This time it was harder to sacrifice but she scraped by. The following year, more fatigued, she was not so welcoming of her boss's praise and pitch for even higher targets again. The fourth year, overwhelmed and dissatisfied, she quit.

In this example, the friend's exertion in the pursuit of one year's goal affects her willingness and/or ability to work hard in the next year, as she progresses from "inspired" to "scraping by" to "fatigued" and ultimately "overwhelmed".<sup>1</sup> Put another way, the cost of her effort (i.e., its negative effect on her utility) spills over from one period to the next; we refer to this as 'effort cost spillover'.

This story also appears to be an example of the broader phenomenon known as 'burnout'. The World Health Organization (2018) defines burnout as "resulting from chronic workplace stress that has not been successfully managed. It is characterized by three dimensions: 1) feelings of energy depletion or exhaustion; 2) increased mental distance from one's job, or feelings of negativism or cynicism related to one's job; and 3) a sense of ineffectiveness and lack of accomplishment." According to a recent survey, "95 percent of HR leaders admit that employee burnout is sabotaging workforce retention". Even among employees who stay with their firms, burnout or fatigue can undermine engagement, sap productivity and fuel absenteeism (Kronos, 2017). Furthermore, the risk of burnout is considered to be particularly high among salespeople, due to their role as 'boundary spanners' who "seek to simultaneously meet the needs of the client and the organization, and who are accountable to the demands of both" (Lewin and Sager, 2007).

Numerous studies (e.g., Cordes and Dougherty, 1993; Maslach et al., 2001; Schaufeli and Bakker, 2004) have found that one of the primary drivers of burnout is a heavy workload (often referred to as "role overload" or "quantitative overload"). The prevalence of burnout is perhaps the most compelling evidence that effort spillover effects not only exist but are common and can be detrimental to both firms and employees.<sup>2</sup> Dimension 1 in the WHO definition (exhaustion) implies an effort cost spillover, as in the above example. Dimension 3 (ineffectiveness) suggests another form of spillover, in which exertion decreases the effectiveness of future effort rather than (or in addition to) increasing its cost. We refer to this as 'productivity spillover'.

An empirical connection between these spillover effects and sales incentives is evident in the findings of Habel et al. (2021) and Babakus et al. (1999). The former finds that an increase in salespeople's variable compensation is associated with not only enhanced sales performance, but also "progressively greater increases in emotional exhaustion". The latter finds that emotional exhaustion has a significant negative impact on salespeople's organizational commitment (among other effects), which increases their intention to leave.

<sup>&</sup>lt;sup>1</sup>The story also illustrates 'target ratcheting', in which the firm responds to an employee's strong performance by raising goals or expectations. Ratcheting has been well studied by researchers in both economics (e.g. Weitzman, 1980; Ickes and Samuelson, 1987; Laffont and Tirole, 1988) and accounting (e.g. Bouwens and Kroos, 2011) and is not the focus of this paper.

<sup>&</sup>lt;sup>2</sup>Positive spillover effects, such as learning and momentum, can also exist. We restrict our attention to cases in which the net spillover effect is negative, as is required for burnout to exist. Analysis of net positive spillovers is left for future research.

The existence of spillover effects implies that salespeople may need to be compensated differently over time, to account for the accumulating effects of their effort. For example, if a salesperson is incentivized to work hard in one period, she may require even stronger incentives in the next period in order to maintain her motivation or to keep her from quitting. Is it optimal for the firm to reduce the salesperson's incentives in the first period, so as to reduce her effort cost in the next? Or is it better to offer strong incentives when the salesperson is 'fresh', then lower them when fatigue sets in? How do spillovers affect the firm's ability to extract surplus?

These questions cannot be answered using the single-period or time-separable models in the incentive design literature, so we employ a dynamic two-period principal-agent model with an agent (salesperson) who experiences effort cost spillover between periods. Specifically, the salesperson's effort cost in the second period is increasing in both her second-period effort *and* her first-period effort. (All of our results are qualitatively similar if we model productivity spillover instead.) We use this model to explore the firm's optimal contract design over time.

According to its formal definition, the term "burnout" is a continuous effect rather than a binary outcome. In informal use, however, it is common to refer to someone as "burned out" specifically when they leave their job as a result of their past effort, as in the above example. To distinguish between these two uses, we will continue to refer to the continuous effects of effort on a salesperson's future utility as "spillover effects" and reserve "burnout" as shorthand for spillover-driven turnover. In particular, we define a burnout equilibrium as one in which the the salesperson exerts so much effort in the first period that she is unwilling to accept any contract that the firm is willing to offer (i.e. that allows the firm to be profitable) in the second period. Put differently, she works so hard initially that in order to continue working, she would require exorbitant remuneration that the firm is unwilling to provide, so she exits the firm. In our analysis, we restrict ourselves to situations in which the first-best outcome – where the maximum long-term expected surplus is generated – requires the salesperson to stay employed and exert positive effort in both periods. We can then examine whether (and why) burnout might occur even when it is sub-optimal.

Standard models tell us that in a single-period setting with a risk-neutral salesperson, the firm can always extract the salesperson's full surplus. The firm can accomplish this by offering her a 'franchise contract' in which she receives zero surplus.<sup>3</sup> With no spillovers, this extends readily to two periods – the firm can extract full surplus by simply replicating the first-period contract offer in the second period. This extension holds whether the firm and salesperson are forward-looking or myopic, and whether or not the firm can commit to future contracts. The simplicity and robustness of the analysis in the absence of spillovers is one reason we will model the salesperson as risk-neutral in our setting.

Now consider what happens with spillovers. First suppose that the firm offers contracts one period at a time. The salesperson anticipates that in period 2, the firm will offer a contract that leaves her with zero expected surplus in that period, just as with no spillovers. We might expect that the firm could still extract full surplus by offering a franchise contract in both periods. After all, with franchise contracts the salesperson should internalize the spillover externality and lower her first-period effort below the single-period optimum on her own. She would indeed do this if she was set on staying with the firm for both periods. However, she has a different option available. She can instead choose to work hard in period 1 (earning a positive surplus)

<sup>&</sup>lt;sup>3</sup>In a franchise contract, the salesperson is paid the net revenue she produces and is charged a fixed 'franchise fee'.

and then quit in period 2, thereby avoiding the increased spillover cost. Thus, offering a pair of franchise contracts will not work.

The firm could instead offer a first-period contract with reduced incentives, to induce the salesperson to temper her period 1 effort. However, with reduced incentives, the salesperson may well choose to work even less than the firm wants in the first period, in order to (secretly) benefit from a lower second-period effort cost. In sharp contrast with the no-spillover case, the firm may be unable to obtain its first-best profits. The first-best outcome is most likely to be achievable when the salesperson's effort cost spillover (and thus, her temptation to shirk) is relatively weak.

When the first-best outcome is not achievable, the firm's equilibrium strategy may be to induce the salesperson to burn herself out. This holds despite our assumptions that she cannot be replaced in the second period and that the maximum profits can only be achieved when she stays with the firm and continues exerting effort. This burnout equilibrium can be quite inefficient, yielding little more than half of the first-best expected profits.

The firm is harmed by the fact that the salesperson anticipates a relatively unattractive second-period contract offer. This obstacle can be surmounted, and the first-best outcome achieved, if the firm is willing and able to commit to an incentive plan for both periods at once. In that case, the firm offers a first-period contract that generally yields the salesperson a negative expected surplus but commits to a second-period contract that makes up for it, with the salesperson earning zero expected surplus across both periods. This commitment need not be a formal, multi-period contract. It could, for instance, be a type of relational contract in which the salesperson trusts the firm to offer a particular second-period contract and the firm reliably does so to avoid 'punishment' by salespeople in the future. Our detailed analysis considers what happens depending on whether the firm can commit to a future contract and whether salespeople are myopic or forward-looking.

This paper offers managerial insights to firms wishing to take a longer-term approach to incentive compensation design. While we focus on the sales context, our model and findings are relevant to any context in which a standard principal-agent model applies and effort cost and/or productivity spillovers exist. In the presence of relatively weak spillovers (or myopic salespeople), firms can achieve their optimal profits by reducing short-term incentives relative to those suggested by existing theory. When spillover effects are strong, firms can achieve their optimal outcomes by committing to future contracts in advance. If such commitment is not feasible, then a burnout strategy of optimizing short-term contracts and letting salespeople exit the firm should be considered.

These results also imply that if a firm is able to reduce the effort spillover effects experienced by its salespeople – perhaps through measures like flexible work arrangements, improved management practices, employee wellness benefits, etc. – it can recognize significant financial gains, in addition to improving its employees' well-being. For example, consider a firm that is currently burning its salespeople out (in equilibrium). If the firm can reduce effort spillovers by enough, it can transition to a first-best equilibrium, with expected profits increasing by as much as 100%. The transition from an inefficient equilibrium to an efficient one means that the gain from the reduction in the spillover cost is disproportionate to the reduction itself.

## 1.1 Literature Review

As noted above, much of the theoretical literature on incentive plan design for workers in the last four decades is based in agency theory. The underlying principal-agent models assume an outcome (e.g. output, sales, profits) that is driven by the effort of the agent, but with a stochastic component, reflecting uncertainty in the production function. While these models have been adapted to a wide range of contexts (e.g. Lal and Staelin, 1986; Joseph and Thevaranjan, 1998; Godes, 2004), they typically consider only a single time period, wherein the principal offers a contract, then the agent chooses whether to accept and, if so, chooses her effort level for the period to which the contract applies.

There is, however, a subset of the compensation literature that considers multiple periods, including a mix of empirical and theoretical research. On the empirical side, Banker et al. (2001) provide a study of performance improvements following the implementation of a "pay-for-performance" compensation plan, using data from multiple periods to explore the relative strength of the "selection effect" and the "effort effect" of incentives. Steenburgh (2008) provides an empirical study of the effects of lump-sum bonuses on salesperson behavior, including the potential for manipulation of order timing. A number of researchers have used multi-period studies to explore the effects of "target ratcheting", the practice of basing targets for one period (at least in part) on performance in the previous period. See Indjejikian et al. (2014) for a review of both theory and empirical research on ratcheting.

Multi-period compensation models have been considered somewhat more extensively in the theoretical literature, but the firm's and agent's utilities are generally assumed to be independent or 'time-separable' across periods (e.g., Mantrala et al., 1997; Gershkov and Perry, 2012; Jerath and Long, 2020). Similarly, dynamic moral hazard models in which decisions are made in continuous time (e.g. Holmström and Milgrom, 1987; Lal and Srinivasan, 1993; Demarzo and Sannikov, 2017) typically assume that preferences remain consistent over time. In our model, the agent's utility – specifically her effort cost – depends on her action in the previous period, introducing an important new dynamic. Schöttner (2017) allows for similar "cost externalities" in a two-period principal-agent model, but the firm only observes outcomes (and determines compensation) at the end of the second period. The agent's effort options are binary and the firm always prefers the higher option in each period. We relax those assumptions, allowing the firm to compensate the agent after each period and considering a range of optimal effort choices. Furthermore, while Schöttner (2017) focuses on whether the firm should offer commission- or bonus-based compensation, we explore how and when the firm can achieve its best possible outcome and the conditions under which it prefers to burn agents out. A small subset of the labor supply literature also considers how workers' effort choices in one period affects their choices in a subsequent period (e.g., Fehr and Goette, 2007; DeJarnette, 2018).

There is also a subset of the dynamic moral hazard literature that focuses on inter-temporal risk sharing and how past sales affect the curvature of the agent's utility function (e.g. Rogerson, 1985; Lambert, 1983). The effect of savings on the optimal incentive schemes have also been analyzed by Fudenberg et al. (1990) and the literature that followed (in particular, the literature that considers hidden savings, such as Ábrahám and Pavoni (2008) and the references therein). Again, in those literatures the agent's cost of effort remains consistent over time, with dynamics affecting her accumulated wealth (and thus utility). In our model, we focus on a risk-neutral agent in order to isolate the cause of inefficiency, so our findings are not affected by the insights from those subsets of the literature. The model that is perhaps most similar to our own is that used by Dearden and Lilien (1990). That paper focuses on "production learning", through which manufacturing costs decline with cumulative volume (i.e., with production experience). When that occurs, firms have an incentive to increase sales, and therefore production, in one period in order to decrease costs in the next. The authors use a two-period model to explore the use of sales force compensation to achieve this objective, optimizing long-term discounted profits in the presence of production learning. Again, however, their model assumes that the agent's utility with respect to income and effort is identical and independent across periods. As a result, their agent does not benefit from a forward-looking strategy, although the firm does. Perhaps not surprisingly, then, their results look quite similar to our own (although inverted, because early effort lowers later costs in their model and raises them in ours) when the agent is assumed to be myopic and the firm forward-looking. However, our model introduces new findings when the agent is forward-looking, adjusting her early effort in anticipation of its effect on her future utility.

Lastly, there is a substantial literature on the subject of employee burnout. The commonly accepted definition in that research is similar to that of the WHO, focusing on three components: emotional exhaustion, depersonalization / cynicism, and diminished personal accomplishment (Cordes and Dougherty, 1993). As noted above, studies have consistently found that "role overload "is a significant driver of emotional exhaustion and consequently burnout. However, workload is typically treated as a fixed characteristic of a job and the link between workload and incentives is rarely considered. One notable exception is Habel et al. (2021), an empirical study of the effects of variable compensation on salesperson health, including emotional exhaustion. Habel et al. find that variable compensation (as a share of total compensation) is positively associated with both performance and emotional exhaustion, which is consistent with our model assumptions. Although Sannikov (2008) devotes one paragraph to situations in which a worker might choose her outside option (in a continuous-time principal-agent setting), ours is believed to be the first use of game theory to study burnout. While most of the existing research focuses on the process of burnout or its three components, we are interested in the relationships between burnout, incentives, and turnover. We treat workload as the result of choices made by the firm and the salesperson, rather than a fixed characteristic.

The remainder of the paper is structured as follows: In the next section, we introduce the model to be analyzed. Next, we derive and discuss the results of the model. Finally, we review the implications of our findings and discuss related research opportunities.

## 2 Model

We analyze a two-period game between a firm and a salesperson. Two periods is the minimal time horizon that allows us to capture the impact of effort cost spillovers on the optimal design of the salesperson's compensation plan and on the firm's performance.

**Salesperson and Firm.** For ease of exposition, sales  $x_t$  in period  $t \in \{1, 2\}$  can be either (relatively) low,  $x^l$ , or high,  $x^h > x^l$ . In each period t, the salesperson chooses an effort level  $e_t \in E = \{a_1, a_2, ..., a_n\} \cup q$ , where  $a_i < a_{i+1} \in \mathbb{R}_+$  and q indicates that the salesperson quits. The minimum effort level  $a_1$  could be zero, indicating no additional effort beyond a baseline amount that the salesperson is happy to exert. Or it could

be the smallest effort that will not trigger action by a supervisor. In any case,  $a_1$  provides value to the firm by generating sales of at least  $x^l$ . In contrast, if at any point the salesperson exits, the firm earns zero from that point on. Thus, we are assuming that the firm is unable to hire and train a replacement within the time frame of the model.<sup>4</sup>

The literature in the standard setting without spillovers often assumes that agents have a continuous effort space, but that assumption leads to equilibrium non-existence issues in our context.<sup>5</sup> There are also good non-technical reasons for using a finite effort choice set. In many contexts the salesperson's effort decision is best modeled as a choice from a discrete set. For instance, a salesperson's effort may be measured in units like customer calls/visits or days on site, where fractional changes are meaningless or have little impact. Or her effort may be predominantly a function of the number of regions or clients she focuses on, so that a set of effort levels with only a few choices, rather than a fine grid with a multitude, may be appropriate. Moreover, the salesperson may perceive changes in her effort as discontinuous jumps, rather than continuous movements. In any case, a continuous strategy space can be approximated in our model by taking the gaps  $a_{i+1} - a_i$  to zero.

When the salesperson does not quit, her effort  $e_t$ , determines the probability of high sales in period  $t, p(e_t) \equiv Pr(x_t = x^h \mid e_t)$ , where p(e) is an increasing function with decreasing increments – that is,  $\frac{p(a_{i+1})-p(a_i)}{a_{i+1}-a_i}$  is decreasing in *i*. This is akin to the usual assumption of concave production in a model with a continuous effort space.

The firm cannot observe the effort the salesperson puts in. It motivates her by basing her pay on sales outcomes, which are mutually observed and contractible. The firm offers contracts  $w_1(x_1)$  in period 1 and  $w_2(x_1, x_2)$  in period 2. Given the binary nature of sales, each contract consists of a pair of payout values  $(l_t, h_t)$  associated with low sales and high sales in that period:

$$w_1(x_1) = \begin{cases} l_1 & \text{if } x_1 = x^l \\ h_1 & \text{if } x_1 = x^h \end{cases} \text{ and } w_2(x_1, x_2) = \begin{cases} l_2(x_1) & \text{if } x_2 = x^l, \text{ given } x_1 \\ h_2(x_1) & \text{if } x_2 = x^h, \text{ given } x_1 \end{cases}$$

The payments in each period can be thought of as a fixed salary  $l_t$  plus a sales-dependent bonus  $h_t - l_t$ . Although the period 2 payments the firm offers can depend on the observed period 1 sales, in pure-strategy equilibria the period 2 optimal contract will typically depend only on period 2 outcomes. In that case, we sometimes write  $w_1 = (l_1, h_1)$  and  $w_2 = (l_2, h_2)$ .

The salesperson receives increasing utility from income and is assumed to be risk-neutral; we use a simple linear utility function u(w) = w. Assuming risk-neutrality makes the model more tractable and transparent, allowing us to isolate the effect of spillovers. In a standard model without spillovers, risk aversion leads to firms offering salespeople reduced incentives and to firms being unable to achieve their first-best outcomes. We obtain both of these features with a risk-neutral salesperson. We impose a limited liability condition on the contract in each period: the salesperson must always receive a non-negative wage, i.e.,  $l_t, h_t \ge 0, t = 1, 2$ . As discussed in Dai and Jerath (2013), pairing limited liability with risk-neutral agents is a simple way to partially model risk aversion, with the agent effectively assumed to be highly averse to downside risk. A limited liability condition is common both in the literature and in practice.

 $<sup>^{4}</sup>$ This is consistent with a finding by the Sales Management Association (Kelly, 2017) that "newly hired salespeople take 11.4 months on average to be successful", which does not include the time it takes to find and hire them.

 $<sup>^{5}</sup>$ As a general matter, the existence of a weak perfect Bayesian equilibrium is not guaranteed in a continuous game.

If the salesperson does not quit, she incurs a cost from the effort she exerts in each period. In period 1 this cost is given by  $c_1(e_1)$ , where  $c_1$  is strictly increasing. As is commonly assumed,  $c_1$  has strictly increasing increments. In period 2, the effort cost is given by  $c_2(e_1, e_2)$ , where i)  $c_2$  is strictly increasing in  $e_2$  and strictly increasing in  $e_1$ , ii) the incremental cost of second-period effort  $e_2$  is strictly increasing, and iii) the incremental cost of second-period effort  $e_1$ .

The role of  $e_1$  in  $c_2(e_1, e_2)$  reflects the spillover effect from the salesperson's earlier effort. The cost spillover could represent a fatigue or burnout effect, as discussed above. It could also represent a saturation effect, in which early-period effort is directed at the easiest tasks or targets – 'low-hanging fruit' – leaving more difficult ones for the later period. Since the firm cannot observe the salesperson's period 1 effort, this implies that it does not directly know the salesperson's period 2 cost function. Nonetheless, the firm can infer period 1 effort (and therefore the cost function) in a pure-strategy equilibrium.

The salesperson's utility from income and disutility from effort are taken to be additively separable, so that her utilities in periods 1 and 2 are given by  $w_1(x_1) - c_1(e_1)$  and  $w_2(x_1, x_2) - c_2(e_1, e_2)$ , respectively. The salesperson maximizes her total expected utility. To streamline the analysis, we make the common assumption that when the salesperson is indifferent between actions, she chooses the one that is best for the firm.

The salesperson's best outside option provides utility  $\overline{U}$  in any single period. If, at the beginning of any period, the firm's (anticipated) contracts yield the salesperson a total expected utility less than her outside option, she rejects the current contract offer, exits the firm, and earns  $\overline{U}$  in each period from that point on.

The firm's profits in each period are defined as net sales minus the salesperson's compensation,  $\pi_t = x_t - w_t$ . The firm maximizes the sum of its expected profits.

The analysis is simplified when limited liability does not bite – that is, when the firm can optimally offer the salesperson a contract with a strictly positive wage in each period. Since positive wages for salespeople are common in most industries, we focus on this case throughout our analysis. Formally, we assume that

$$\bar{U} > \max_{e \in E} \left\{ p(e) \left( x^h - x^l \right) - c_1(e) \right\}$$
(1)

which guarantees that equilibrium wages are strictly positive.

#### Sequence of Events and Equilibrium Concept. The sequence of events is as follows:

- 1. The firm offers the salesperson a contract  $w_1(x_1)$ .
- 2. The salesperson chooses whether to accept the contract. If she refuses, she quits the firm and the game ends. If she accepts, she chooses her effort level  $e_1$ .
- 3. Sales  $x_1$  are realized and the firm pays the salesperson  $w_1(x_1)$ .
- 4. If the salesperson accepted contract  $w_1$ , the firm offers her a contract  $w_2(x_1, x_2)$ .
- 5. The salesperson chooses whether to accept the contract. If she refuses, she quits the firm and the game ends. If she accepts, she chooses her effort level  $e_2$ .
- 6. Sales  $x_2$  are realized and the firm pays the salesperson  $w_2(x_1, x_2)$ .

We also consider a variation of this sequence in which the firm commits to contracts for both periods at the outset, while the salesperson remains uncommitted.

- 1. The firm offers a pair of contracts  $w_1(x_1)$  and  $w_2(x_1, x_2)$ .
- 2. The salesperson chooses whether to accept the contract  $w_1$ . If she refuses, she quits the firm and the game ends. If she accepts, she chooses her effort level  $e_1$ .
- 3. Sales  $x_1$  are realized and the firm pays the salesperson  $w_1(x_1)$ .
- 4. The salesperson chooses whether to continue under contract  $w_2$ . If she refuses, she quits the firm and the game ends. If she accepts, she chooses her effort level  $e_2$ .
- 5. Sales  $x_2$  are realized and the firm pays the salesperson  $w_2(x_1, x_2)$ .

The equilibrium concept we use is weak perfect Bayesian equilibrium. None of our results rely on assuming specific out-of-equilibrium beliefs.

## 3 Analysis and Results

We solve the model using backward induction. Given a period 2 contract offer, the worker chooses her effort level to maximize her period 2 utility in light of her period 1 effort level  $e_1$ . That is, she chooses  $\hat{e}_2 = \arg \max E [w_2 - c_2 (e_1, e_2) | e_1]$ .<sup>6</sup> She accepts the contract if and only if  $E [w_2 - c_2 (e_1, \hat{e}_2) | e_1] \ge \overline{U}$ .

Consider a pure strategy equilibrium in which, along the equilibrium path, the worker picks effort  $\hat{e}_1$  in period 1. The firm does not observe this effort but, in equilibrium, it designs the period 2 contract under the assumption that the worker picked effort  $\hat{e}_1$ . Thus, the firm chooses the period 2 contract to maximize  $E[x_2 - w_2 | \hat{e}_1, \hat{e}_2]$ , subject to the constraint  $E[w_2 - c_2(e_1, \hat{e}_2) | \hat{e}_1, \hat{e}_2] \ge \overline{U}$ . If the constraint cannot be satisfied with a non-negative expected payoff for the firm, it instead offers a contract that always pays 0, ensuring that the worker quits.

Given a period 1 contract offer, the worker chooses period 1 effort to maximize  $E[w_1 - c_1(e_1) + w_2 - c_2(e_1, e_2)]$ , anticipating the contract that the firm will offer in period 2 and her own response to it. If this does not achieve  $2\overline{U}$  over both periods, the worker rejects the contract. The firm designs the period 1 contract that maximizes  $E[x_1 - w_1 + x_2 - w_2]$ , anticipating the worker's period 1 response and the continuation game.

Given any effort levels  $e_1$  and  $e_2$ , denote the period 1 expected surplus of the salesperson and firm combined by  $E[S_1(e_1)] \equiv E[x_1 | e_1] - c_1(e_1) - \overline{U}$ , the period 2 expected surplus by  $E[S_2(e_1, e_2)] \equiv E[x_2 | e_2] - c_2(e_1, e_2) - \overline{U}$ , and the total two-period expected surplus by  $E[S(e_1, e_2)] \equiv E[S_1(e_1)] + E[S_2(e_1, e_2)]$ . We assume throughout that both players do not discount the future; slight discounting would not qualitatively affect our results.

Let  $e_1^*$  and  $e_2^*$  be the period 1 and period 2 effort levels that maximize the total two-period expected surplus. For ease of exposition, we take these to be unique and refer to them as first-best efforts.

<sup>&</sup>lt;sup>6</sup>Recall that if there are several maximands, the worker chooses the effort level that is best for the firm. We call that effort level  $\hat{e}_2$ , and, somewhat abusively, write  $\hat{e}_2 = \arg \max E [w_2 - c_2 (e_1, e_2) | e_1]$ .

The firm's first-best outcome occurs when the salesperson exerts  $e_1^*$  and  $e_2^*$  and the firm extracts all of the expected surplus. Thus, the first-best outcome yields the firm expected profits of  $E[S(e_1^*, e_2^*)]$  and the salesperson an expected surplus of 0.

It is not readily apparent whether the first-best outcome requires more effort from the salesperson in the first period or the second. Put another way, is it better for the salesperson to push herself early then ease up later when effort is more costly, or to hold back early to minimize the cost spillover before making a late push? In fact, both scenarios are possible, as we will see in Section 3.2.

Consider the effort choice in E that maximizes period 1 expected surplus. We denote this effort by  $e_1^m$ , since it is the period 1 *myopic* optimal level, and we also assume it to be unique. Clearly,  $e_1^* \leq e_1^m$  due to effort cost spillovers. If spillovers are very weak, then  $e_1^* = e_1^m$ ; if spillovers are very strong,  $e_1^* = a_1$ . In both of these cases, the strategic analysis essentially reduces to a one-period problem, which is not our interest. Hence, we restrict our attention to the case  $a_1 < e_1^* < e_1^m$ .<sup>7</sup>

• Assumption:  $a_1 < e_1^* < e_1^m$ .

Since it is optimal for the salesperson to exert effort less than  $e_1^m$  in period 1, it must be optimal for her to remain employed in period 2. Given any period 1 effort  $e_1$ , let  $e_2^m(e_1)$  be the effort level  $e_2$  that maximizes period 2 expected surplus  $E[S_2(e_1, e_2)]$ . The following lemma is immediate.

**Lemma 1** 1)  $e_2^* \neq q$ . 2)  $e_2^m(e_1)$  is weakly decreasing in  $e_1$ .

(All proofs are in the appendix.)

Since  $e_2^* \neq q$ , it is not optimal for the salesperson to burn out. Nonetheless, if she (suboptimally) exerts too much effort in the first period, the spillover cost from that effort might make it unprofitable to employ her in the second period. That is, there may exist a first-period effort threshold such that if the salesperson exceeds it, then she is unwilling to accept *any* contract that the firm is willing to offer in the second period. We call this the "burnout threshold",  $e_1^b$ , formally defined by:

$$e_1^b \equiv \min \left\{ e_1 \in E : E[x_2 \mid e_2] - c_2(e_1, e_2) - \bar{U} \le 0, \forall e_2 \in E \right\}$$

Suppose  $e_1^b$  exists. If  $e_1 > e_1^b$ , then the firm cannot profitably employ the salesperson in period 2. Since  $e_2^* \neq q$ , we have  $e_1^* \leq e_1^b$ .

If an equilibrium exists in which the salesperson's first-period effort exceeds the burnout threshold  $(e_1 > e_1^b)$ , we refer to it as a burnout equilibrium.

## 3.1 No Spillovers

We begin with the standard model without spillovers, where  $c_2(e_1, e_2) \equiv c_2(e_2) = c_1(e_2)$  and the first-best effort levels are  $e_1^* = e_2^* = e_1^m$ . This model is stationary in character, so the fact that it is typically written for a single period is insignificant. It is well established that the firm can obtain its first-best outcome by, in each period, offering the salesperson a franchise contract that leaves her with no expected surplus. Specifically, the firm can offer the contracts

$$w_t = x_t - \left( E[x_1 \mid e_1^m] - \bar{U} - c_1(e_1^m) \right), \ t = 1, 2$$
(2)

 $<sup>^7</sup>$ Dropping this assumption presents no technical difficulties but leads to extra (uninteresting) cases to analyze.

The salesperson responds by choosing the efficient effort level in both periods. Note that the firm obtains its first-best outcome whether the players are myopic or forward-looking, and whether or not the firm can commit to future contracts. The situation with spillovers is quite different. The first-best outcome is no longer certain and these model assumptions have important ramifications.

## 3.2 Myopia

A natural hypothesis to explain salespeople over-exerting themselves, and sometimes burning out, is that firms and salespeople are myopic – they ignore the impact of first-period effort on second-period costs and simply maximize period-by-period. Thus, in period 1 the firm offers strong incentives – for instance, contract (2) – and the salesperson responds by putting in effort  $e_1^m > e_1^*$ . If  $e_1^m > e_1^b$ , the salesperson burns out and quits in the second period. In any case, the firm earns less than its first-best.

Consider the following example with spillovers.

**Example 1** Let  $E = [0, .01, .02, ..., 1] \cup q$ ,  $c_1(e_1) = e_1^2$ ,  $c_2(e_1, e_2) = (1 + ae_1)e_2^2 + be_1$ , p(e) = e,  $x^l = 1, x^h = 2$ , and  $\overline{U} = \frac{1}{2}$ . A simple calculation shows  $e_1^m = \frac{1}{2}$ . Moreover, i) a = 1,  $b = 0 \implies e_1^* = 0.44$  and  $e_2^* = 0.35$ , while ii)  $a = 0, b = \frac{3}{5} \implies e_1^* = 0.2$  and  $e_2^* = 0.5$ .

As a side remark, this example shows that, with spillovers, first-best effort levels can be increasing, as in i), or decreasing, as in ii) (constant is also possible).

- Under both i) and ii), a myopic firm optimizes with the period 1 contract (l<sub>1</sub>, h<sub>1</sub>) = (<sup>1</sup>/<sub>4</sub>, <sup>5</sup>/<sub>4</sub>), to which a myopic salesperson responds with a period 1 effort choice <sup>1</sup>/<sub>2</sub> = e<sup>m</sup><sub>1</sub> > e<sup>\*</sup><sub>1</sub>.
  - When a = 0 and  $b = \frac{3}{5}$ , the salesperson burns out and quits in period 2.
  - When a = 1 and b = 0, the salesperson does not burn out. In period 2, the firm optimizes with a contract  $(l_2, h_2) = (\frac{1}{3}, \frac{4}{3})$  and the salesperson chooses  $e_2 = \frac{1}{3} < e_2^*$ .

The (banal) hypothesis that firms and salespeople are myopic may well explain some cases of overexertion but surely not all.

Suppose instead that the firm, with all its experience, is forward-looking, while the less-experienced salesperson naïvely fails to anticipate the draining effect effort has across time and acts myopically. Now, the firm can achieve its first-best outcome. In period 1, it offers weakened incentives that make  $e_1^*$  the salesperson's myopic best response. That is, the firm offers a period 1 contract  $(l_1, h_1)$  such that  $e_1^* = \arg \max_{e \in E} [p(e) h_1 + (1 - p(e)) l_1 - c_1(e)]$ . This contract must have reduced incentives  $(h_1 - l_1)$  relative to the no-spillover contract, so that the salesperson does not over-exert herself (avoiding a choice of  $e_1^m > e_1^*$ ). In period 2, the firm can elicit the effort  $e_2^*$  with a franchise contract.

• In the above example, when a = 1 and b = 0, a forward-looking firm optimizes with the first-period contract  $(l_1, h_1) = (0.31, 1.19)$  and a myopic salesperson chooses effort level  $e_1^*$ . Note that  $h_1 - l_1 = 0.88 < 1$ , the bonus with no spillovers.

We have the following proposition.

**Proposition 1** When the firm and the salesperson are both myopic, the salesperson exerts strictly more than the first-best effort in period 1 and weakly less than the first-best effort in period 2. Thus, the firm earns strictly less than the first-best expected profit. If spillovers are sufficiently large, the result is a burnout equilibrium, with the salesperson exiting in period 2.

When only the salesperson is myopic, the firm earns its first-best expected payoff. The salesperson's period 1 contract includes weaker incentives (i.e., smaller  $h_1 - l_1$ ) than it does when the firm is myopic. Burnout is never an equilibrium result.

## 3.3 Forward-Looking Firm and Salesperson

Now suppose that the salesperson also anticipates the interperiod effect of effort. That is, both the firm and the salesperson maximize their two-period surplus while accounting for effort spillovers. This can be considered full rationality.<sup>8</sup> It is now important to consider whether the firm can commit to a period 2 contract at the time that it offers a period 1 contract. If the firm can commit to a future contract, then it always obtains its first-best outcome; if the firm cannot commit, this is no longer assured. We begin with the commitment case.

#### 3.3.1 Commitment.

With full rationality, the firm could attempt to obtain its first-best outcome in a manner that parallels the no-spillovers case but which takes account of spillovers. That is, the firm could offer franchise contracts that yield the salesperson zero expected surplus in each period when the salesperson chooses the efficient levels  $e_1^*, e_2^*$ . These two contracts are

$$w_1(x_1) = x_1 - \left( E[x_1 \mid e_1^*] - \bar{U} - c_1(e_1^*) \right)$$
$$w_2(x_1, x_2) = x_2 - \left( E[x_2 \mid e_2^*] - \bar{U} - c_2(e_1^*, e_2^*) \right)$$

If the salesperson were unable to quit, these contracts would achieve the first-best outcome for the firm. The salesperson would internalize the externality and maximize by choosing  $e_1^*, e_2^*$ , while the firm recouped the salesperson's surplus in fixed transfers.

However, the salesperson has the option of quitting at any time. With the above contracts, the salesperson could do better by choosing  $e_1^m > e_1^*$  in period 1, which gives her a positive expected surplus, then quitting in period 2. Hence, this attempt would fail to achieve first-best.

There are two alternative routes the firm could take to achieve its first-best outcome.

The first is to offer a contract that gives the salesperson a negative expected surplus in the first period and a positive one in the second, so that the salesperson does not quit in period 2. For instance, the firm can offer the following two franchise contracts

$$w_1(x_1) = x_1 - \left( E[x_1 \mid e_1^m] - \bar{U} - c_1(e_1^m) \right)$$
(3)

$$w_2(x_1, x_2) = x_2 - \left( E[x_2 \mid e_2^*] - \bar{U} - c_2(e_1^*, e_2^*) + E[x_1 \mid e_1^*] - c_1(e_1^*) - E[x_1 \mid e_1^m] + c_1(e_1^m) \right)$$
(4)

<sup>&</sup>lt;sup>8</sup>We omit the analysis of the somewhat odd case of a myopic firm and forward-looking salesperson. In that setting, the myopic firm does not generally obtain its first-best outcome.

Observe that  $w_1$  here is the same contract as when the firm and salesperson are both myopic. Now, however, the salesperson is forward-looking and does not exert effort  $e_1^m$  in period 1. Instead, she internalizes the externality of the spillover and exerts effort  $e_1^* < e_1^m$ , with a negative expected surplus of  $E[x_1 | e_1^*] - c_1(e_1^*) - (E[x_1 | e_1^m] - c_1(e_1^m))$ . The firm then 'reimburses' her for that loss in period 2, leaving her with zero net surplus. If she chooses  $e_1^m$  in the first period and then quits, she again obtains zero expected surplus. Any other effort combination yields a negative expected surplus across both periods. Thus, this pair of contracts achieves the first-best outcome.

Notice, however, that once period 2 arrives the firm would prefer to deviate from its proposed secondperiod contract and instead offer one that yields the salesperson zero expected surplus in period 2. For instance, the firm could profitably deviate to the offer:

$$w_2(x_1, x_2) = x_2 - \left( E[x_2 \mid e_2^*] - \bar{U} - c_2(e_1^*, e_2^*) \right)$$

Thus, the pair of contracts (3) and (4) can work only if the firm is willing and able to credibly commit to a period 2 contract in period 1.

Suppose that the firm can commit to a period 2 contract in period 1. That is, at the beginning of period 1 the firm can offer a binding period 2 contract (along with the period 1 contract). The salesperson is free to quit the firm at any time. Then we have the following result:

**Proposition 2** When both the firm and the salesperson are forward-looking, and the firm can commit to a future contract, the firm obtains its first-best outcome.

In practice, there are a variety of reasons why firms might hesitate to formally commit to future sales incentive contracts. For example, firms that use quota-based contracts typically prefer to finalize their quotas as late as possible, in order to ensure that they reflect the best possible information about products, territories, customers, etc. However, commitment need not be through a formal future contract offer. In effect, the equilibrium relies on the salesperson *believing* that the 'correct' contract will be offered in the second period and the firm being sufficiently motivated not to deviate from that offer. For example, although not explicitly represented in our two-period model, that motivation could be the result of the firm's longerterm concerns about its reputation and ability to recruit and retain salespeople. Such 'relational contracts' rely on trust between the salesperson and the firm. This trust may or may not exist (or be sufficiently strong), depending on the firm's culture, past behavior, relations between salespeople and management, etc.

#### 3.3.2 No Commitment.

If the firm cannot, or will not, commit to a future contract, the only potential route to a first-best outcome is for the firm to offer a pair of contracts that yield the salesperson zero expected surplus in each period. Since the salesperson receives no surplus in either period, effort level  $e_1^*$  must be her myopic best response. Otherwise, she would prefer to exert her myopic optimal effort in the first period, then exit in the second.

For  $e_1^*$  to be a myopic best response, the firm must offer the salesperson relatively weak incentives in period 1 (compared to contract (3), for example). However, the salesperson is then tempted to reduce her period 1 effort even more than the firm would like, in order to lower her period 2 costs (unbeknownst to the firm). Since the period 1 loss from reducing her effort is relatively small due to the weak incentives, it may well be impossible to find a pair of contracts that yield the salesperson zero in each period and make  $e_1^*$  an optimal long-term choice.<sup>9</sup> In other words, the firm may be unable to offer first-period incentives that are both weak enough to prevent overexertion and strong enough to prevent shirking. This difficulty is less pronounced when spillovers are smaller, as that decreases the salesperson's period 2 gain from shirking in period 1.

**Proposition 3** Suppose the firm cannot commit to a future contract. When both the firm and the salesperson are forward-looking, the firm obtains its first-best outcome if and only if shirking (reducing period 1 effort below  $e_1^*$ ) has a sufficiently small impact on the salesperson's effort cost spillover.

Formally, let  $e_1^* = a_g$  and  $e_2^* = a_k$ . The firm can obtain its first-best outcome if and only if for all  $i \leq g$ and  $j \geq k$ ,

$$c_{1}(a_{g}) - c_{1}(a_{i}) - (p(a_{g}) - p(a_{i})) \frac{c_{1}(a_{g+1}) - c_{1}(a_{g})}{p(a_{g+1}) - p(a_{g})} + c(a_{g}, a_{k}) - c_{2}(a_{i}, a_{j}) - (p(a_{k}) - p(a_{j})) \frac{c(a_{g}, a_{k}) - c(a_{g}, a_{k-1})}{p(a_{k}) - p(a_{k-1})} \leq 0$$
(5)

This result stands in sharp contrast to Proposition 2 and to the no-spillovers case, with or without commitment.

Clearly, the salesperson never prefers to exert *more* than the myopic best response to any contract. However, she might prefer to shirk in period 1 in order to reduce her effort cost in period 2. Therefore, in order for a first-best equilibrium to exist, the firm must be able to ensure that the salesperson's loss from shirking always outweighs her subsequent benefit. The first line of condition (5) represents the salesperson's period 1 loss from shirking (maximized by the firm's contract design), while the second line represents the benefit she realizes in period 2 (minimized by design). When the condition holds, the firm can ensure that the salesperson's net benefit from deviating from  $e_1^*, e_2^*$  is always negative.

Condition (5) implies that a first-best equilibrium exists if and only if the reduction in effort cost spillover is sufficiently small for each effort level  $a_i < a_g = e_1^*$  (i.e.,  $c_2(a_g, a_k) - c_2(a_i, a_k)$  sufficiently small). To see this, it helps to consider the simplest deviation, in which the salesperson shirks by exerting  $a_i < a_g$  in period 1 and her optimal choice remains  $a_k = e_2^*$  in period 2. In that case, condition (5) simplifies to

$$\underbrace{(p(a_g) - p(a_i)) \frac{c_1(a_{g+1}) - c_1(a_g)}{p(a_{g+1}) - p(a_g)}}_{\text{expected payout loss in period 1}} - \underbrace{[c_1(a_g) - c_1(a_i)]}_{\text{cost saving in period 1}} \ge \underbrace{c(a_g, a_k) - c(a_i, a_k)}_{\text{cost saving in period 2 (spillover)}}$$
(6)

The left-hand side of this inequality represents the net cost to the salesperson of deviating to  $a_i$ , given bonus  $h_1 - l_1 = \frac{c_1(a_{g+1}) - c_1(a_g)}{p(a_{g+1}) - p(a_g)}$ . As shown in Appendix B,  $a_g$  is a myopic best response in that case and  $a_i < a_g$  is not, so that cost must be strictly positive. Therefore, as spillover savings go to 0 (i.e. as  $c(a_i, a_k)$ goes to  $c(a_g, a_k)$ ), the condition is clearly satisfied and the firm can achieve its first-best outcome. On the other hand, it appears that the condition must fail when spillover savings are large. There is an upper bound on  $c(a_g, a_k) - c(a_i, a_k)$  that results from the definition of  $(a_g, a_k)$  as first-best efforts (implying that  $(a_g, a_k)$  is more efficient than  $(a_i, a_k)$ ). However, it can be shown that the condition does indeed fail as  $c(a_g, a_k) - c(a_i, a_k)$  approaches that bound from below.

<sup>&</sup>lt;sup>9</sup>In fact, it will be impossible when the salesperson's strategy space is fine enough, since her first-period loss from a marginal reduction in effort below the myopic optimal then approaches zero, while the second-period gain from the reduced cost spillover remains positive.

Proposition 3 indicates that when a firm cannot pre-commit to contracts, its first-best outcome may be unachievable. Next, we find that when the firm cannot achieve its first-best (i.e., when condition (5) fails), the unique equilibrium outcome may be burnout. Thus, although the first-best outcome requires effort from the salesperson in both periods, the firm may purposefully over-incentivize her in the first period, optimizing her short-term effort despite knowing that she will then exit in the second.

**Proposition 4** When the firm cannot achieve its first-best outcome, it sometimes offers a burnout contract in period 1. The resulting inefficiency can be severe: The firm's expected payoff under a burnout equilibrium can approach half of the first-best, although it can never be lower.

Propositions 3 and 4 combine to suggest that some firms have an opportunity to increase profits significantly by investing to decrease effort cost spillovers (perhaps through popular initiatives like wellness programs, flexible work arrangements, etc.), but that small changes may not have any impact at all when spillovers are high. In particular, when there is a burnout equilibrium, which implies that spillover savings from shirking are high (by Proposition 3), then a marginal reduction has no impact because salespeople exit before spillovers take effect. However, if the firm can reduce spillovers to the point that condition (5) is satisfied, then the burnout equilibrium is replaced by a first-best equilibrium which can increase profits substantially (by Proposition 4).

Two Examples. Below, we present two examples that illustrate Propositions 3 and 4.

In both examples, we allow the salesperson's effort to take one of three values  $(a_1, a_2, a_3)$ , we assign  $x^l = 500, x^h = 1300$ , and  $\overline{U} = 500$ , and the probabilities of success and first-period costs are described by the following matrix:

Period 1 effort 
$$p(a_i) c_1(a_i)$$
  
 $a_1 \frac{1}{100} 1$   
 $a_2 \frac{52}{100} 2$   
 $a_3 \frac{58}{100} 4$ 
(7)

In the first example, condition (5) is satisfied, so the firm can obtain its first-best outcome.

**Example 2** The probabilities of success and the first-period effort costs are given by matrix (7). Secondperiod effort costs  $c_2(a_i, a_j)$  are given by the matrix:

Period 2 effort $\rightarrow$	$a_1$	$a_2$	$a_3$
$\textit{Period 1 effort} \downarrow$	$c_2\left(a_i,a_1\right)$	$c_2\left(a_i,a_2\right)$	$c_2\left(a_i,a_3\right)$
$a_1$	1	2	51
$a_2$	2	17	72
$a_3$	300	450	500

It is easily verified that the first-best effort choices are  $(e_1^*, e_2^*) = (a_2, a_2)$ . Moreover, these are also the unique equilibrium effort levels. These efforts are implemented by the contract  $(l_1, h_1) = (\frac{1454}{3}, 518)$ , followed by  $(l_2, h_2) = (101, 901)$ . The firm earns the first-best surplus 813.

See Appendix F for a proof that these strategies constitute an equilibrium.

The next example illustrates Proposition 4 – condition (5) is not satisfied and the firm cannot obtain its first-best outcome. It differs from Example 2 in that the cost spillover from period 1 effort has increased. (For simplicity, the only change is an increase in the cost of effort level  $a_2$  in period 2 following  $a_2$  in period 1.)

**Example 3** The probabilities of success and the first-period effort costs are given by matrix (7). Secondperiod effort costs  $c_2(a_i, a_j)$  are given by the matrix:

$Period \ 2 \ effort \rightarrow$	$a_1$	$a_2$	$a_3$
$\textit{Period 1 effort} \downarrow$	$c_2\left(a_i,a_1\right)$	$c_2\left(a_i,a_2\right)$	$c_2\left(a_i,a_3\right)$
$a_1$	1	2	51
$a_2$	2	22	72
$a_3$	300	450	500

Again,  $(e_1^*, e_2^*) = (a_2, a_2)$  but now these effort levels cannot be achieved in equilibrium. Instead, the unique equilibrium outcome is an effort of  $a_3$  in period 1, resulting in the salesperson burning out and quitting. This is implemented by contract offers  $(l_1, h_1) = (\frac{1454}{3}, 518)$  (as in Example 2) and  $(l_2, h_2) = (0, 0)$ . The firm earns a profit of 460, which is 57% of the first-best profit 808.

Why is it that first-best is no longer achievable? The firm could, as before, offer a contract that makes effort  $a_2$  myopically optimal in period 1, followed by a period 2 contract in which  $h - l = x^h - x^l$ . But now, the increased spillover cost means that the (forward-looking) salesperson would prefer to shirk and exert  $a_1$ in period 1, sacrificing some of her initial payoff in favor of a reduction in her later effort cost.

See Appendix E for a proof that the strategies in Example 3 constitute an equilibrium. Note that the equilibrium effort levels are constant in Example 2 and decreasing in Example 3. It is also possible for them to be increasing. Note also that the firm in Example 3 can move from a burnout equilibrium to a first-best equilibrium, and increase expected profits by over 75%, if it can somehow decrease the salesperson's effort cost spillover following action  $a_2$ .

While Proposition 4 focuses on burnout, there can be other types of non-first-best equilibria when condition (5) fails. One possibility is an equilibrium in which the salesperson mixes between  $e_1^*$  and some lower action  $a_i < e_1^*$ . In such an equilibrium, the contract offer in period 2 is conditional on the sales outcome in period 1. For example, when sales are low in period 1, the firm offers a contract with a smaller salary in period 2.

The likelihood of a burnout equilibrium may be affected by additional factors that are not included in our model. For example, although we have assumed that a salesperson who exits cannot be replaced withing the model timeline, it is possible that a firm could hire and deploy a new salesperson quickly enough to achieve some productivity in period 2. In that case, burning salespeople out would become even more attractive, increasing the likelihood of a burnout equilibrium. On the other hand, we have not included additional costs associated with turnover (e.g., relationship disruption, hiring and training new salespeople), which would make burnout less attractive and thus less likely as an equilibrium outcome.

While we have assumed that the salesperson is risk-neutral in order to isolate the effects of effort cost spillovers, our main qualitative results, including the firm's inability to achieve its first-best outcome and the existence of burnout equilibria, will continue to hold with some risk aversion. We leave the complete analysis of a risk-averse agent for future research.

## 4 Conclusion

Employee burnout is a significant issue for firms and employers across a wide range of industries and has gained increasing attention in recent years. The prevalence of burnout is an indication that the impacts of work-related effort (such as fatigue) on effort costs and/or productivity are incurred by salespeople not only *while* they are working, but for some time after. We incorporate an effort cost spillover into a two-period principal-agent model to study its impact on the optimal design of incentive contracts over time. We focus on the sales force context due to the particular susceptibility of salespeople to burnout and the lack of observability of most sales effort.

The standard principal-agent model typically used to study sales force incentive design cannot account for these dynamic effort cost spillover effects. Applying a repeated single-period model in the presence of spillovers results in the firm over-incentivizing the salesperson in the first period. The salesperson then exerts more than the first-best effort in the first period and less in the second, while the firm obtains less than its first-best profits.

Using a dynamic model, we find that a forward-looking firm employing a myopic salesperson *can* achieve its first-best profits. It does so by offering weaker incentives in the first period than it would without spillovers, thus inducing the salesperson to optimally reduce her first-period effort.

When the firm and the salesperson are both forward-looking, the analysis is not quite as straightforward. If the firm can commit to a future contract in advance, it can always achieve its first-best outcome. Otherwise, it may not be able to. In particular, the first-best outcome is achievable without contract commitment when the spillover effect is sufficiently small.

When the firm is unable or unwilling to commit to a future contract and the first-best cannot be achieved, the firm's best possible outcome can be a burnout equilibrium, in which the firm knowingly incentivizes the salesperson to exert so much effort in the first period that she quits in the second. This can be the case even when the resulting profits are little more than half of what would result if the firm could induce the salesperson to stay and exert the first-best effort in both periods.

These results have a number of managerial implications. In order to maximize profits over time, firms should consider the long-term effects of effort on their salespeople's well-being, willingness to work, and productivity and should adjust expectations and incentives accordingly. The existence of burnout equilibria might help to explain the prevalence of burnout, even when firms can seemingly benefit by increasing retention. For firms seeking to reduce burnout, our results suggest that they might do so by committing to longer-term incentive contracts. Similarly, a policymaker seeking to decrease burnout rates should consider interventions that encourage or enable firms to commit to longer-term contracts. Alternatively, burnout can be reduced if firms are able to lessen the effort cost spillovers experienced by their employees. This might help to explain the growing popularity of benefits such as employee wellness programs and flexible work arrangements, among others.

We conclude with a brief discussion of some of our model assumptions and limitations, which suggest

opportunities for further research. We have made the simplifying assumption that a salesperson's outside option utility in the second period is unaffected by her first-period effort. However, if her effort cost spillover represents fatigue, then she might continue to feel that effect after leaving and joining another firm, potentially lowering her outside option utility. Conversely, if the spillover reflects task saturation (i.e. exhausting the 'low-hanging fruit' of sales opportunities), then instead of affecting the salesperson's outside option, it might continue to affect the firm and the salesperson's replacement after she exits. It would also be interesting, both theoretically and managerially, to consider how our results would be affected if the spillover cost of effort was subject to some exogenous shock, forcing the firm to design the second-period contract with imperfect information about the salesperson's effort cost. Finally, this research is motivated by common concerns about employee burnout, so we have focused on positive effort cost spillover, with the salesperson's second-period effort cost increasing in her first-period effort. It might also be worth considering the effects of negative spillovers, which could exist if early effort generates some type of learning or momentum effect.

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## Appendix

## A Proof of Lemma 1

For part 1), note that if  $e_2^* = q$ ,  $e_1^*$  maximizes  $E[S_1(e_1)]$ . But that is impossible, as we have assumed that  $e_1^m$  is the maximizer, and  $e_1^m \neq e_1^*$ .

For (2), let  $e_2^m(e)$  maximize  $E[S_2(e, e_2)] = p(e_2)x^h + (1 - p(e_2))x^l - c_2(e, e_2) - \overline{U}$ . Let e' > e, and suppose  $e_2^m(e') \equiv e_2' > e_2^m(e) \equiv e_2$ . Then we have:

$$p(e'_{2})(x^{h} - x^{l}) - c_{2}(e', e'_{2}) - (p(e_{2})(x^{h} - x^{l}) - c_{2}(e', e_{2})) > 0 >$$

$$p(e'_{2})(x^{h} - x^{l}) - c_{2}(e, e'_{2}) - (p(e_{2})(x^{h} - x^{l}) - c_{2}(e, e_{2}))$$

$$\Rightarrow c_{2}(e', e'_{2}) - c_{2}(e', e_{2}) < c_{2}(e, e'_{2}) - c_{2}(e, e_{2})$$

which contradicts assumption (iii), that the incremental cost of second period effort  $e_2$  is increasing in first-period effort  $e_1$ . Therefore,  $e_2^m(e_1)$  is weakly decreasing in  $e_1$ .

## **B** Proof of Proposition 1

When both the firm and the salesperson are myopic, in period 1 the firm will implement  $e_1^m > e_1^*$ . Then, by Lemma 1,  $e_2^m(e_1^m) \le e_2^m(e_1^*) = e_2^*$ . Since  $e_1^*$  and  $e_2^*$  are unique, and the salesperson obtains  $\overline{U}$  in both periods, the firm obtains less than the first best profits.

Now, take any model that satisfies our assumptions  $e_1^m > e_1^*$  and  $e_1^b > e_1^*$ :

If  $e_1^m \ge e_1^b$ , then burnout results (and spillovers must be large, as  $e_1^b$  is infinitely large with no spillovers). If  $e_1^b > e_1^m$ , then there exists another model with larger spillovers such that for all  $e > e_1^m$ ,  $c_2(e, e_2^m(e)) > E(x | e, e_2^m(e)) - \overline{U}$ . In other words, there is no  $e > e_1^m$  for which the subject can be profitably employed. Therefore,  $e_1^m \ge \tilde{e}_1^b$  (the new burnout threshold) and the subject burns out. Notice that these changes in  $c_2$ do not affect  $e_1^m$  (which concerns only the first period surplus), or  $(e_1^*, e_2^*)$  as  $c_2(e_1^*, e_2^*)$  is unaffected because  $e_1^m > e_1^*$  so  $c_2(e_1^*, e)$  is unaffected for all e. Thus, when both the firm and the salesperson are myopic and spillovers are sufficiently large, burnout results.

Next, we turn to the case in which the firm is forward-looking. In order to implement  $e_1^*$  and extract all surplus in period 1, the firm sets  $l_1$  such that

$$p(e_1^*) h_1 + (1 - p(e_1^*)) l_1 - c_1(e_1^*) = \bar{U} \Leftrightarrow l_1 = \bar{U} + c_1(e_1^*) - p(e_1^*) (h_1 - l_1)$$

and chooses  $h_1 - l_1$  so that  $e_1^*$  is the salesperson's best response:  $\forall e \in E$ 

$$p(e_{1}^{*})h_{1} + (1 - p(e_{1}^{*}))l_{1} - c_{1}(e_{1}^{*}) = p(e_{1}^{*})(h_{1} - l_{1}) - c_{1}(e_{1}^{*}) + l_{1} \ge p(e)(h_{1} - l_{1}) - c_{1}(e) + l_{1} \Leftrightarrow (p(e_{1}^{*}) - p(e))(h_{1} - l_{1}) \ge c_{1}(e_{1}^{*}) - c_{1}(e) \Leftrightarrow \forall e' > e_{1}^{*} > e, \quad \frac{c_{1}(e') - c_{1}(e_{1}^{*})}{p(e') - p(e_{1}^{*})} \ge h_{1} - l_{1} \ge \frac{c_{1}(e_{1}^{*}) - c_{1}(e)}{p(e_{1}^{*}) - p(e)}$$

$$(8)$$

The interval is non-empty because increments are increasing for c and decreasing for p. By offering the following contract in the second period, the firm can induce the salesperson to exert  $e_2^*$  and can extract all surplus:

$$(l,h) = \left(c_2\left(e_1^*, e_2^*\right) + \bar{U} - p\left(e_2^*\right)\left(x^h - x^l\right), c_2\left(e_1^*, e_2^*\right) + \bar{U} + \left(1 - p\left(e_2^*\right)\right)\left(x^h - x^l\right)\right)\right)$$

Thus, the firm can induce  $e_1^*$  and  $e_2^*$  and extract all surplus, achieving the first-best outcome.

We now show that first-period incentives  $h_1 - l_1$  are weaker than when the firm is myopic,  $h_1^m - l_1^m$ . If the firm is myopic, it offers first period incentives such that for all  $e \leq e_1^m$ ,

$$h_1^m - l_1^m \ge \frac{c_1(e_1^m) - c_1(e)}{p(e_1^m) - p(e)}.$$

In particular, because  $e_1^* < e_1^m$ , and because of (8),

$$h_1^m - l_1^m \ge \frac{c_1(e_1^m) - c_1(e_1^*)}{p(e_1^m) - p(e_1^*)} \ge h_1 - l_1$$

showing the weaker incentives.

Burnout does not result, as the first-best payoffs obtained by the firm preclude the salesperson quitting.

## C Proof of Proposition 2

Contracts (3) and (4) implement  $(e_1^*, e_2^*)$  and give the full surplus to the firm, as the salesperson maximizes

$$E[x_1 | e_1] - c_1(e_1) + E[x_2 | e_1] - c_2(e_1, e_2)$$

by choosing  $(e_1^*, e_2^*)$  and obtains  $2\overline{U}$  across both periods.

## D Proof of Proposition 3

**Sufficiency**. We will construct  $(h_1^*, l_1^*), (h_2^*, l_2^*)$  that implement  $a_g$  followed by  $a_k$ . From the proof of Proposition 1, we have that if  $e'' \ge e \ge e'$ , then e is a better (myopic) response than e' or e'' to a contract (l, h) iff

$$\frac{c_1(e'') - c_1(e)}{p(e'') - p(e)} \ge h - l \ge \frac{c_1(e) - c_1(e')}{p(e) - p(e')}$$
(9)

Define the first-period contract  $w_1 = (l_1, h_1)$  by

$$l_{1}^{*} = \frac{p(a_{g+1})c_{1}(a_{g}) - c_{1}(a_{g+1})p(a_{g})}{p(a_{g+1}) - p(a_{g})} + \bar{U}, h_{1}^{*} = \frac{(1 - p(a_{g}))c_{1}(a_{g+1}) - c_{1}(a_{g})(1 - p(a_{g+1}))}{p(a_{g+1}) - p(a_{g})} + \bar{U} \quad (10)$$

Under the above contract,  $h_1^* - l_1^* = \frac{c_1(a_{g+1}) - c_1(a_g)}{p(a_{g+1}) - p(a_g)}$ , so the salesperson is indifferent between  $a_g$  and  $a_{g+1}$  (in the first period). Therefore,  $e_1^* = a_g$  is a myopic optimal effort choice, and she has the least possible incentive to deviate to a lower effort. The contract also makes the salesperson's participation constraint bind at that effort, ensuring that the firm collects all expected surplus in period 1.

Because  $\frac{c_1(a_{i+1})-c_1(a_i)}{p(a_{i+1})-p(a_i)}$  is increasing in *i*, and because (9) holds for  $e'' = a_{g+1}$ ,  $e = a_g$  and  $e' = a_{g-1}$ , the salesperson has no short-term incentive to deviate in period 1. Deviating to an action  $a_i > a_g$  in period 1 (and then optimizing in period 2) is not profitable: it pays weakly less in period 1 and increases the salesperson's effort cost in period 2. So the only potentially profitable deviations are to  $a_i < a_g$ . Below we establish that if a < a', the utility-maximizing effort in period 2 after *a* is weakly larger than after *a'* for any contract (h, l). Since in period 2 the salesperson will choose the utility-maximizing effort conditional on  $a_i$ we only need to consider period 2 actions  $a_j \ge a_k$  in period 2. The most the firm can punish such a deviation to  $a_j \ge a_k$  in period 2 is by picking  $h_2 - l_2$  as small as possible in the following range (which ensures that  $a_k$  is the salesperson's optimal choice after  $a_g$ ) so that deviating to  $a_j \ge a_k$  is least attractive.

$$\frac{c(a_g, a_j) - c(a_g, a_k)}{p(a_j) - p(a_k)} \ge h_2 - l_2 \ge \frac{c(a_g, a_k) - c(a_g, a_{k-1})}{p(a_k) - p(a_{k-1})}$$
(11)

Adding the constraint that the salesperson's expected utility from  $a_k$  (following  $a_g$ ) is  $\overline{U}$ , we obtain the target period 2 contracts.

$$h_{2}^{*} - l_{2}^{*} = \frac{c(a_{g}, a_{k}) - c(a_{g}, a_{k-1})}{p(a_{k}) - p(a_{k-1})} \\ p(a_{k}) h_{2}^{*} + (1 - p(a_{k})) l_{2}^{*} - c(a_{g}, a_{k}) = \bar{U} \end{cases} \xrightarrow{l_{2}^{*}} l_{2}^{*} = \bar{U} + c(a_{g}, a_{k}) - p(a_{k}) \frac{c(a_{g}, a_{k}) - c(a_{g}, a_{k-1})}{p(a_{k}) - p(a_{k-1})} \\ h_{2}^{*} = \bar{U} + c(a_{g}, a_{k}) + (1 - p(a_{k})) \frac{c(a_{g}, a_{k}) - c(a_{g}, a_{k-1})}{p(a_{k}) - p(a_{k-1})}$$

$$(12)$$

For the firm to achieve its first-best outcome, it must be able to ensure that the salesperson's loss from any shirking in period 1 outweighs her resulting increased efficiency in period 2. Under the contracts above, this holds for a deviation to  $(a_i, a_j)$  if

$$p(a_{i})h_{1}^{*} + (1 - p(a_{i}))l_{1}^{*} - c_{1}(a_{i}) + p(a_{j})h_{2}^{*} + (1 - p(a_{j}))l_{2}^{*} - c(a_{i}, a_{j}) \leq 2U \Leftrightarrow$$
(13)  

$$p(a_{i})\frac{c_{1}(a_{g+1}) - c_{1}(a_{g})}{p(a_{g+1}) - p(a_{g})} + \frac{p(a_{g+1})c_{1}(a_{g}) - c_{1}(a_{g+1})p(a_{g})}{p(a_{g+1}) - p(a_{g})} - c_{1}(a_{i}) +$$
( $p(a_{j}) - p(a_{k})$ ) $\frac{c(a_{g}, a_{k}) - c(a_{g}, a_{k-1})}{p(a_{k}) - p(a_{k-1})} + c(a_{g}, a_{k}) - c(a_{i}, a_{j}) \leq 0 \Leftrightarrow$ (13)  

$$c_{1}(a_{g}) - c_{1}(a_{i}) - (p(a_{g}) - p(a_{i}))\frac{c_{1}(a_{g+1}) - c_{1}(a_{g})}{p(a_{g+1}) - p(a_{g})} +$$
( $p(a_{j}) - p(a_{k})$ ) $\frac{c(a_{g}, a_{k}) - c(a_{g}, a_{k-1})}{p(a_{k}) - p(a_{k-1})} + c(a_{g}, a_{k}) - c(a_{i}, a_{j}) \leq 0$ (14)

which is exactly condition (5). Thus, condition (5) ensures that the salesperson cannot benefit from deviating.

Let us now show that if a < a', the utility-maximizing effort in period 2 after a is weakly larger than after a' for any contract (h, l). This ensures that if an individual deviates to  $a_i < a_g$  in period 1, one only needs to consider deviations to  $a_j \ge a_k$  in period 2. We will do this by showing that if it is beneficial in period 2 for the salesperson to increase her action after a', then it is also beneficial to increase it after a < a': for actions b and b' > b, we have  $c(a, b') - c(a, b) \le c(a', b') - c(a', b)$  so that

$$\left. \begin{array}{l} p\left(b'\right)h + \left(1 - p\left(b'\right)\right)l - c\left(a',b'\right) \ge p\left(b\right)h + \left(1 - p\left(b\right)\right)l - c\left(a',b\right) \Leftrightarrow \\ \left(p\left(b'\right) - p\left(b\right)\right)\left(h - l\right) \ge c\left(a',b'\right) - c\left(a',b\right) \\ \end{array} \right\} \\ \Rightarrow \quad \left\{ \begin{array}{l} \left(p\left(b'\right) - p\left(b\right)\right)\left(h - l\right) \ge c\left(a,b'\right) - c\left(a,b\right) \Leftrightarrow \\ p\left(b'\right)h + \left(1 - p\left(b'\right)\right)l - c\left(a,b'\right) \ge p\left(b\right)h + \left(1 - p\left(b\right)\right)l - c\left(a,b\right) \end{array} \right\} \right. \end{array} \right\}$$

we know then that increasing from b to b' is profitable if and only if  $(p(b') - p(b))(h - l) \ge c(a', b') - c(a', b)$ , and since we also know that  $c(a, b') - c(a, b) \le c(a', b') - c(a', b)$ , we obtain

$$\begin{cases} (p(b') - p(b))(h - l) \ge c(a', b') - c(a', b) \\ c(a, b') - c(a, b) \le c(a', b') - c(a', b) \end{cases} \Rightarrow (p(b') - p(b))(h - l) \ge c(a, b') - c(a, b) \Rightarrow \\ p(b')h + (1 - p(b'))l - c(a, b') \ge p(b)h + (1 - p(b))l - c(a, b) \end{cases}$$

as was to be shown.

**Necessity.** Assume condition (5) is violated and suppose a pair of contracts  $(h_1^e, l_1^e)$  and  $(h_2^e, l_2^e)$  constitute an equilibrium that implements  $a_g$  followed by  $a_k$  (if an equilibrium exists in which the second-period contract is contingent on the first-period outcome, then there must also exist an equilibrium in which it is not contingent, so we focus on the latter). In period 2 we know that  $h_2^e, l_2^e$  must satisfy (11), and that the utility of the salesperson in period 2 must be  $\bar{U}$  (if the equilibrium implements  $a_g$  in period 1 the firm can extract the full surplus in period 2). Given this, the first period contract must be such that  $a_g$  is a myopic best response, and therefore  $(h_1^e, l_1^e)$  must satisfy (9) with  $e = a_g$ .

The following are equal, but in particular:

$$\begin{split} p\left(a_{g}\right)h_{1}^{e} + \left(1 - p\left(a_{g}\right)\right)l_{1}^{e} - c_{1}\left(a_{g}\right) + p\left(a_{k}\right)h_{2}^{e} + \left(1 - p\left(a_{k}\right)\right)l_{2}^{e} - c\left(a_{g}, a_{k}\right) \\ &\geq p\left(a_{g}\right)h_{1}^{*} + \left(1 - p\left(a_{g}\right)\right)l_{1}^{*} - c_{1}\left(a_{g}\right) + p\left(a_{k}\right)h_{2}^{*} + \left(1 - p\left(a_{k}\right)\right)l_{2}^{*} - c\left(a_{g}, a_{k}\right) \Leftrightarrow \\ p\left(a_{g}\right)h_{1}^{e} + \left(1 - p\left(a_{g}\right)\right)l_{1}^{e} + p\left(a_{k}\right)h_{2}^{e} + \left(1 - p\left(a_{k}\right)\right)l_{2}^{e} \\ &\geq p\left(a_{g}\right)h_{1}^{*} + \left(1 - p\left(a_{g}\right)\right)l_{1}^{*} + p\left(a_{k}\right)h_{2}^{*} + \left(1 - p\left(a_{k}\right)\right)l_{2}^{*} \Leftrightarrow \\ p\left(a_{k}\right)\left[\left(h_{2}^{e} - l_{2}^{e}\right) - \left(h_{2}^{*} - l_{2}^{*}\right)\right] - p\left(a_{g}\right)\left[\left(h_{1}^{*} - l_{1}^{*}\right) - \left(h_{1}^{e} - l_{1}^{e}\right)\right] \\ &\geq \left(l_{1}^{*} - l_{1}^{e}\right) + \left(l_{2}^{*} - l_{2}^{e}\right) \end{split}$$

Then, since

$$\begin{aligned} h_1^* - l_1^* &\geq h_1^e - l_1^e \\ h_2^e - l_2^e &\geq h_2^* - l_2^* \\ p\left(a_i\right) &\leq p\left(a_g\right) \left(\text{since } a_i \leq a_g\right) \\ p\left(a_j\right) &\geq p\left(a_k\right) \left(\text{since } a_j \geq a_k\right) \end{aligned}$$

we obtain

$$\begin{split} p\left(a_{j}\right)\left[\left(h_{2}^{e}-l_{2}^{e}\right)-\left(h_{2}^{*}-l_{2}^{*}\right)\right]-p\left(a_{i}\right)\left[\left(h_{1}^{*}-l_{1}^{*}\right)-\left(h_{1}^{e}-l_{1}^{e}\right)\right]\\ &\geq p\left(a_{k}\right)\left[\left(h_{2}^{e}-l_{2}^{e}\right)-\left(h_{2}^{*}-l_{2}^{*}\right)\right]-p\left(a_{g}\right)\left[\left(h_{1}^{*}-l_{1}^{*}\right)-\left(h_{1}^{e}-l_{1}^{e}\right)\right]\geq\left(l_{1}^{*}-l_{1}^{e}\right)+\left(l_{2}^{*}-l_{2}^{e}\right)\Rightarrow\\ p\left(a_{i}\right)h_{1}^{e}+\left(1-p\left(a_{i}\right)\right)l_{1}^{e}-c_{1}\left(a_{i}\right)+p\left(a_{j}\right)h_{2}^{e}+\left(1-p\left(a_{j}\right)\right)l_{2}^{e}-c\left(a_{i},a_{j}\right)\\ &\geq p\left(a_{i}\right)h_{1}^{*}+\left(1-p\left(a_{i}\right)\right)l_{1}^{*}-c_{1}\left(a_{i}\right)+p\left(a_{j}\right)h_{2}^{*}+\left(1-p\left(a_{j}\right)\right)l_{2}^{*}-c\left(a_{i},a_{j}\right)>2\bar{U} \end{split}$$

Where the strict inequality follows from the fact that if (5) is violated, the utility of  $a_i$  followed by  $a_j$ , under contracts  $(h_1^*, l_1^*), (h_2^*, l_2^*)$  is strictly greater than  $2\overline{U}$  (from equations 13 and 14, which is just 5). Therefore, the salesperson can earn a strictly positive surplus by deviating to  $a_i, a_j$  which contradicts  $(h_1^e, l_1^e)$ and  $(h_2^e, l_2^e)$  as an equilibrium that implements  $a_g$  followed by  $a_k$ .

## E Proof of Proposition 4

The existence (and potential inefficiency) of burnout equilibria are shown in the following example.

Suppose the salesperson's effort takes one of three values,  $E = (a_1, a_2, a_3)$ , which is the simplest case that displays a range of outcomes. Outputs and reservation utility are  $x^h = 1300$ ,  $x^l = \bar{U} = 500$ , and the

probabilities of high output and the first-period costs under each effort are given by the following matrix:

Period 1 effort 
$$p(a_i) \equiv p_i \quad c_1(a_i) \equiv c_{a_i}$$
  
 $a_1 \quad \frac{1}{100} \quad 1$   
 $a_2 \quad \frac{52}{100} \quad 2$   
 $a_3 \quad \frac{58}{100} \quad 4$ 
(15)

Second-period effort costs  $c_2(a_i, a_j)$  are given by the matrix:

Period 2 effort $\rightarrow$	$a_1$	$a_2$	$a_3$
Period 1 effort $\downarrow$	$c_{a_i a_1}$	$c_{a_i a_2}$	$c_{a_i a_3}$
$a_1$	1	2	51
$a_2$	2	22	72
$a_3$	300	450	500

The following is an equilibrium.

A. In period 1, the firm offers a contract  $(l_1, h_1) = (\frac{1454}{3}, 518)$ . Thus,  $h_1 - l_1 \ge \frac{100}{3} = \frac{c_{a_3} - c_{a_2}}{p_3 - p_2}$  and participation holds with equality  $(\frac{58}{100}h_1 + \frac{42}{100}l_1 - 4 = 500)$ .

B. In period 2:

- 1. in subgames in which the period 1 offer was such that  $h l \ge \frac{100}{3}$ , the firm offers a contract  $(l_2, h_2) = (0, 0)$
- 2. in subgames in which  $\frac{100}{3} > h l \ge \frac{40}{17} = \frac{c_{a_2} c_{a_1} + p_1(c_{a_2a_2} c_{a_1a_2})}{p_2 p_1}$  and period 1 output was high, the firm offers  $\binom{l_1^H}{2}, h_2^H = (106, 906)$ . (This provides a salesperson who played  $a_2$  in period 1 with a maximum of 0 surplus in period 2.)
- 3. in subgames in which  $\frac{100}{3} > h l \ge \frac{40}{17}$  and period 1 output was low, the firm offers  $(l_2^H, h_2^H)$  (from the previous point) with probability  $q = \frac{17}{660} (h l) \frac{2}{33} \in [0, \frac{79}{99})$ , and  $(l_2^L, h_2^L) = (86, 886)$  with probability 1 q. (This provides a salesperson who played  $a_1$  in period 1 with a maximum of 0 surplus in period 2.)
- 4. in subgames in which  $\frac{40}{17} > h l \ge \frac{100}{51} = \frac{c_{a_2} c_{a_1}}{p_2 p_1}$  and output was high, the firm offers  $(l_2^H, h_2^H)$  with probability  $p = \frac{51}{20} (h l) 5 \in [0, 1)$  and  $(l_2^L, h_2^L) = (86, 886)$  with probability 1 p.
- 5. in subgames in which  $\frac{40}{17} > h l \ge \frac{100}{51}$  and output was low, the firm offers  $(l_2^L, h_2^L)$ .
- 6. in subgames in which  $\frac{100}{51} > h l \ge 0$  the firm offers  $(l_2^L, h_2^L)$ .
- C. In period 1, the salesperson:
  - 1. plays  $a_3$  if  $h-l \ge \frac{100}{3}$  (and expected surplus is non-negative this also applies to all points below; otherwise she quits).
  - 2. plays  $a_2$  with probability  $\pi_T = \frac{b(1-p_1)}{1-bp_1-(1-b)p_2} = \frac{165}{1741}$  for  $b = 1 \frac{800p_2-c_{a_2a_2}}{800p_2-c_{a_1a_2}} = \frac{10}{207}$  and  $a_1$  with probability  $1 \pi_T$  if  $\frac{100}{3} > h l \ge \frac{40}{17}$  (where T indicates that this is the Top part of the interval where  $a_2$  could be optimal).

- 3. plays  $a_2$  with probability  $\pi_B = \frac{bp_1}{bp_1 + (1-b)p_2} = \frac{5}{5127}$  and  $a_1$  with probability  $1 \pi_B$  if  $\frac{40}{17} > h l \ge \frac{100}{51}$ .
- 4. plays  $a_1$  if  $\frac{100}{51} > h l$ .
- D. After exerting effort  $i \in \{a_1, a_2, a_3\}$  in period 1, the salesperson quits after any offer (l, h) in period 2 that yields utility  $\max_{j \in \{a_1, a_2, a_3\}} p_j h + (1 p_j) l c_{ij} < \overline{U}$ . Otherwise, she accepts and exerts any effort in  $\arg \max_{j \in \{a_1, a_2, a_3\}} p_j h + (1 p_j) l c_{ij}$ .

To check this, start backwards. Clearly, (D) describes an optimal choice by the salesperson in the last step.

Now, consider (B.1): If the period 1 offer had  $h - l \ge \frac{100}{3}$ , then the salesperson chose  $a_3$  according to (C.1). In period 2, then, expected surplus is negative for all actions  $j \in \{a_1, a_2, a_3\}$   $(\frac{1}{100}1300 + \frac{99}{100}500 - 300 < 500 = \overline{U}, \frac{52}{100}1300 + \frac{48}{100}500 - 450 < 500, \frac{58}{100}1300 + \frac{42}{100}500 - 500 < 500)$ , so offering (0,0) is a best response by the firm. Also, in the subgame in which the firm offers  $h - l \ge \frac{100}{3}$  and will offer a contract (0,0) in the second period, exerting  $a_3$  is a best response by the salesperson, as second-period surplus will be 0 and in the first period

$$\begin{split} u_{a_3} &= \frac{58}{100}h + \frac{42}{100}l - 4 \ge \frac{52}{100}h + \frac{48}{100}l - 2 = u_{a_2} \Leftrightarrow h - l \ge \frac{100}{3}\\ u_{a_3} &\ge \frac{1}{100}h + \frac{99}{100}l - 1 = u_{a_1} \Leftrightarrow h - l \ge \frac{100}{19}. \end{split}$$

In general, for  $a_3$  to be optimal when expected period 2 surplus will be 0, the condition is

$$p_{3}h + (1 - p_{3})l - c_{a_{3}} \geq p_{2}h + (1 - p_{2})l - c_{a_{2}} \Leftrightarrow h - l \geq \frac{c_{a_{3}} - c_{a_{2}}}{p_{3} - p_{2}}.$$

$$p_{3}h + (1 - p_{3})l - c_{a_{3}} \geq p_{1}h + (1 - p_{1})l - c_{a_{1}}$$
(16)

The former condition implies the latter because

$$(16) \Leftrightarrow h - l \ge \frac{c_{a_3} - c_{a_2}}{p_3 - p_2} \\ \frac{c_{a_3} - c_{a_2}}{p_3 - p_2} \ge \frac{c_{a_2} - c_{a_1}}{p_2 - p_1} \\ \end{cases} \Rightarrow h - l \ge \frac{c_{a_2} - c_{a_1}}{p_2 - p_1} \Leftrightarrow p_2 h + (1 - p_2) l - c_{a_2} \ge p_1 h + (1 - p_1) l - c_{a_1}.$$

So in any subgame with  $h - l \ge \frac{100}{3}$  the strategies described are an equilibrium.

Next, consider (**B.2**): In the period 1 subgame, the firm offered  $\frac{100}{3} > h - l \ge \frac{40}{17}$  and the salesperson chose  $a_2$  with probability  $\pi_T = \frac{165}{1741}$  and  $a_1$  with probability  $1 - \pi_T = \frac{1576}{1741}$ . Then, if output was high, the firm's beliefs that the salesperson chose  $a_2$  are

$$P\left(a_{2} \mid x^{h}, \pi_{T}\right) = \frac{p_{2}\pi_{T}}{p_{2}\pi_{T} + p_{1}\left(1 - \pi_{T}\right)}$$

$$= \frac{\frac{52}{100}\frac{1576}{1741}}{\frac{52}{100}\frac{1576}{1741} + \frac{1}{100}\left(1 - \frac{1576}{1741}\right)} = \frac{81\,952}{82\,117} > \frac{10}{207} = b.$$

$$(17)$$

After exerting either  $a_1$  or  $a_2$  in period 1,  $a_2$  maximizes the second-period surplus  $S_2 = x^l + p(x^h - x^l) - c - \bar{U} = p(x^h - x^l) - c$ , as follows:

$$\begin{split} S_2^{a_1 a_2} &= \frac{52}{100} 800 - 2 = 414 > 413 = \frac{58}{100} 800 - 51 = S_2^{a_1 a_3} \\ &> 7 = \frac{1}{100} 800 - 1 = S_2^{a_1 a_1} \\ S_2^{a_2 a_2} &= \frac{52}{100} 800 - 22 = 394 > 392 = \frac{58}{100} 800 - 72 = S_2^{a_2 a_3} \\ &> 6 = \frac{1}{100} 800 - 2 = S_2^{a_2 a_1} \end{split}$$

so the firm will offer a contract that implements  $a_2$ , but must decide whether to induce the salesperson to accept if she chose  $a_2$  in period 1. If so (in which case, she will also accept following  $a_1$ ), the optimal contract extracts all surplus following  $a_2$ . Therefore,

$$(l_2^H, h_2^H) = (x^l - S_2^{a_2 a_2}, x^h - S_2^{a_2 a_2}) = (106, 906);$$

Otherwise, if she will only accept following  $a_1$ , then the optimal contract extracts all surplus, so

$$(l_2^L, h_2^L) = (x^l - S_2^{a_1 a_2} + \bar{U}, x^h - S_2^{a_1 a_2}) = (86, 886)$$

Before we proceed, note that a salesperson who chose  $a_1$  in period 1 and faces contract  $(l_2^H, h_2^H)$  chooses  $a_2$  in period 2 and earns expected utility

$$Z = p_2 \left( x^h - S_2^{a_2 a_2} \right) + (1 - p_2) \left( x^l - S_2^{a_2 a_2} \right) - c_{a_1 a_2} = p_2 x^h + (1 - p_2) x^l - c_{a_1 a_2} - S_2^{a_2 a_2}$$
  
$$= p_2 x^h + (1 - p_2) x^l - c_{a_1 a_2} - \left( p_2 \left( x^h - x^l \right) - c_{a_2 a_2} \right) = x^l + c_{a_2 a_2} - c_{a_1 a_2}$$
(18)

To see that offering  $(l_2^H, h_2^H)$  is optimal after high output, note that earning  $S_2^{a_2a_2}$  with certainty is better than receiving  $S_2^{a_1a_2}$  with probability  $1 - P(a_2 | x^h, \pi)$  (i.e., the firm prefers to offer a more-generous contract that the salesperson will always accept over a less-generous one that she will accept only after exerting  $a_1$ ):

$$S_2^{a_2 a_2} \ge \left(1 - P\left(a_2 \mid x^h, \pi\right)\right) S_2^{a_1 a_2} \Leftrightarrow P\left(a_2 \mid x^h, \pi\right) \ge 1 - \frac{S_2^{a_2 a_2}}{S_2^{a_1 a_2}} = \frac{10}{207} \equiv b.$$
(19)

Therefore, it is optimal for the firm to offer the contract  $(l_2^H, h_2^H)$ . We will see after analyzing (B.3) that it is optimal for the salesperson to mix.

Next, consider (B.3). After low output, the firm's belief that the salesperson played  $a_2$  is

$$P(a_2 | x^l, \pi_T) = \frac{(1-p_2)\pi_T}{(1-p_2)\pi_T + (1-p_1)(1-\pi_T)}$$

$$= \frac{(1-\frac{52}{100})\frac{165}{1741}}{(1-\frac{52}{100})\frac{165}{1741} + (1-\frac{1}{100})(1-\frac{165}{1741})} = \frac{10}{207}$$
(20)

Thus, from (19), the firm is indifferent between offering  $(l_2^H, h_2^H)$  (and having the salesperson accept after both  $a_2$  and  $a_1$ ) and offering  $(l_2^L, h_2^L)$  (and having her accept only after  $a_1$ ), and both are optimal. We can now check that the firm's strategy (offer  $(l_2^H, h_2^H)$  after high output, and offer  $(l_2^H, h_2^H)$  with probability q and  $(l_2^L, h_2^L)$  with probability 1 - q after low output) makes randomizing between  $a_2$  and  $a_1$  an optimal strategy for the salesperson in period 1. Exerting  $a_2$  yields

$$\begin{aligned} u_{a_2a_2} &= \frac{52}{100} \left( h + \frac{52}{100} h_2^H + \frac{48}{100} l_2^H - 22 \right) + \frac{48}{100} \left( l + q \left( \frac{52}{100} h_2^H + \frac{48}{100} l_2^H - 22 \right) + (1 - q) \bar{U} \right) - 2 \\ &= \frac{52}{100} \left( h + \frac{52}{100} 906 + \frac{48}{100} 106 - 22 \right) + \frac{48}{100} \left( l + q \left( \frac{52}{100} 906 + \frac{48}{100} 106 - 22 \right) + (1 - q) 500 \right) - 2 \\ &= \frac{12}{25} l + \frac{13}{25} h + 498 \end{aligned}$$

(The last equality actually holds for any q, as  $(l_2^H, h_2^H)$  and quitting both result in  $\overline{U}$ .) Playing  $a_1$  then  $a_2$  yields

$$\begin{split} u_{a_1a_2} &= \frac{1}{100} \left( h + \frac{52}{100} h_2^H + \frac{48}{100} l_2^H - 2 \right) + \frac{99}{100} \left( l + q \left( \frac{52}{100} h_2^H + \frac{48}{100} l_2^H - 2 \right) + (1 - q) \left( \frac{52}{100} h_2^L + \frac{48}{100} l_2^L - 2 \right) \right) - 1 \\ &= \frac{1}{100} \left( h + \frac{52}{100} 906 + \frac{48}{100} 106 - 2 \right) + \frac{99}{100} \left( l + q \left( \frac{52}{100} 906 + \frac{48}{100} 106 - 2 \right) + (1 - q) \left( \frac{52}{100} 886 + \frac{48}{100} 86 - 2 \right) \right) - 1 \\ &= \frac{12}{25} l + \frac{13}{25} h + 498 \end{split}$$

1

(Here we need to use the equilibrium  $q = \frac{17}{660} (h - l) - \frac{2}{33}$  to obtain the last equality.) More generally, playing  $a_1, a_2$  yields

$$u_{a_1 a_2} = p_1 \left( h_1 + Z \right) + \left( 1 - p_1 \right) \left( l_1 + qZ + \left( 1 - q \right) \bar{U} \right) - c_{a_1}$$

so, using  $Z = x^l + c_{a_2 a_2} - c_{a_1 a_2} \equiv x^l + d = \bar{U} + d$  and  $q = \frac{(h_1 - l_1)(p_2 - p_1) - c_{a_2} + c_{a_1}}{d(1 - p_1)} - \frac{p_1}{(1 - p_1)}$ , we obtain

$$u_{a_1a_2} = p_1 (h_1 + U + d) + (1 - p_1) (l_1 + q (U + d) + (1 - q) U) - c_{a_1}$$
  
=  $p_2h_1 + (1 - p_2) l_1 + \overline{U} - c_{a_2} = u_{a_2a_2}$ 

So indeed both  $a_2$  and  $a_1$  are optimal in the first period for the salesperson, and she is indifferent randomizing (both actions are better than playing  $a_3$ , as  $h-l < \frac{100}{3}$ ). Then the strategies described are an equilibrium in subgames following contract offers with  $\frac{100}{3} > h-l \ge \frac{40}{17}$ . More generally, as we will use it later, the condition to check whether p (the probability of offering  $(l_2^H, h_2^H)$  after high output) and q make the salesperson indifferent is

$$u_{a_{2}a_{2}} = p_{2}h_{1} + (1 - p_{2})l_{1} - c_{a_{2}} + \bar{U} = p_{1}\left(h_{1} + pZ + (1 - p)\bar{U}\right) + (1 - p_{1})\left(l_{1} + qZ + (1 - q)\bar{U}\right) - c_{a_{1}} = u_{a_{1}a_{2}}$$
(21)

We now turn to (**B.4**), which applies when the contract offer was  $\frac{40}{17} > h - l \ge \frac{100}{51}$  and output was high in period 1. In that case, the strategy of the firm is to offer  $(l_2^H, h_2^H)$  with probability  $p = \frac{51}{20}(h - l) - 5$ and  $(l_2^L, h_2^L)$  with 1 - p in period 2. From (C.3), the salesperson exerted  $a_2$  with probability  $\pi_B = \frac{5}{5127}$  and  $a_1$  with  $1 - \pi_B$  in period 1. Thus, after high output  $x^h$ , the firm's belief that the salesperson exerted  $a_2$  is given by replacing  $\pi_T$  with  $\pi_B$  in equation (17):

$$P\left(a_2 \mid x^h, \pi_B\right) = \frac{p_2 \pi_B}{p_2 \pi_B + p_1 \left(1 - \pi_B\right)} = \frac{\frac{52}{100} \frac{5}{5127}}{\frac{52}{100} \frac{5}{5127} + \frac{1}{100} \left(1 - \frac{5}{5127}\right)} = \frac{10}{207} = k$$

Therefore, the firm is indifferent between offering the two contracts (as established in (19)). To check that the salesperson wants to mix in the first period, we must first check (B.5).

(B.5) applies when  $\frac{40}{17} > h - l \ge \frac{100}{51}$  and output was low in period 1. The firm then offers  $(l_2^L, h_2^L)$  in period 2. From (C.3), again, the salesperson exerted  $a_2$  with probability  $\pi_B$  and  $a_1$  with  $1 - \pi_B$  in period 1. The firm's belief that the salesperson played  $a_2$  is given by replacing  $\pi_T$  with  $\pi_B$  in equation (20):

$$P\left(a_2 \mid x^l, \pi_B\right) = \frac{\left(1 - p_2\right)\pi_B}{\left(1 - p_2\right)\pi_B + \left(1 - p_1\right)\left(1 - \pi_B\right)} = \frac{\left(1 - \frac{52}{100}\right)\frac{5}{5127}}{\left(1 - \frac{52}{100}\right)\frac{5}{5127} + \left(1 - \frac{1}{100}\right)\left(1 - \frac{5}{5127}\right)} = \frac{40}{84553} < b$$

so offering only  $(l_2^L, h_2^L)$  is optimal.

Next, we show that the salesperson's mixing strategy is optimal given the firm's strategy. (Recall: After  $x^h$ , the firm offers  $(l_2^H, h_2^H)$  with probability  $p = \frac{(h-l)(p_2-p_1)-c_{a_2}+c_{a_1}}{(c_{a_2a_2}-c_{a_1a_2})p_1} = \frac{51}{20}(h-l)-5$  and  $(l_2^L, h_2^L)$  otherwise; after  $x^l$ , the firm always offers  $(l_2^L, h_2^L)$ .) In particular, we show that  $a_2$  and  $a_1$  yield the same expected utility. From equation (21) (with q = 0) we obtain the desired equality:

$$u_{a_{1}a_{2}} = p_{1} \left( h + pZ + (1-p)\bar{U} \right) + (1-p_{1}) \left( l + \bar{U} \right) - c_{a_{1}}$$
  
$$= p_{1} \left( h + \frac{(h-l)(p_{2}-p_{1}) - c_{a_{2}} + c_{a_{1}}}{dp_{1}} d + \bar{U} \right) + (1-p_{1}) \left( l + \bar{U} \right) - c_{a_{1}}$$
  
$$= p_{2}h + (1-p_{2}) l - c_{a_{2}} + \bar{U} = u_{a_{2}a_{2}}$$

Finally, consider (B.6), which applies when  $\frac{100}{51} > h - l \ge 0$ . In that case, exerting  $a_1$  is optimal for the salesperson, as reflected in (C.4), because her expected utility in period 2 will be  $\overline{U}$  after any period 1 action, and in period 1:

$$\begin{array}{rcl} u_{a_1} & = & \displaystyle \frac{1}{100}h + \frac{99}{100}l - 1 > \displaystyle \frac{52}{100}h - \frac{48}{100}l - 2 = u_{a_2} \Leftrightarrow h - l < \displaystyle \frac{100}{51} = \displaystyle \frac{c_{a_2} - c_{a_1}}{p_2 - p_1} \\ u_{a_1} & = & \displaystyle \frac{1}{100}h + \displaystyle \frac{99}{100}l - 1 > \displaystyle \frac{56}{100}h - \displaystyle \frac{44}{100}l - 4 = u_{a_3} \Leftrightarrow h - l < \displaystyle \frac{60}{11} = \displaystyle \frac{c_{a_3} - c_{a_1}}{p_3 - p_1}. \end{array}$$

Since the firm knows that the salesperson exerts  $a_1$  in equilibrium,  $(l_2^L, h_2^L)$  is optimal in period 2.

We now calculate the expected profits in the subgame determined by each period 1 offer of (l, h):

• From (B.1) and (C.1), if  $h - l \ge \frac{100}{3}$ , then the salesperson exerts  $a_3$  then quits, so an optimal contract in that range satisfies participation with equality,  $\frac{58}{100}h + \frac{42}{100}l - 4 = 500$ . Any such contract yields the same expected profits:

$$V_{B1} = \frac{58}{100} (1300 - h) + \frac{42}{100} (500 - l) = \frac{58}{100} 1300 + \frac{42}{100} 500 - 504 = 460.$$

• In (B.2) and (B.3), an optimal period 1 contract satisfies participation with equality for  $a_2$  (or  $a_1$ , as the salesperson is indifferent):  $\frac{52}{100}h + \frac{48}{100}l - 2 = 500 \Leftrightarrow l = \frac{6275}{6} - \frac{13}{12}h$ . Expected profits are (Note: q vanishes, so no need to substitute):

$$\begin{aligned} V_{B23} = \pi_T \left[ p_2 \left( x^h - h + S_2^{a_2 a_2} \right) + (1 - p_2) \left( x^l - l + q S_2^{a_2 a_2} + (1 - q) 0 \right) \right] + \\ & \left( 1 - \pi^T \right) \left[ p_1 \left( x^h - h + S_2^{a_2 a_2} \right) + (1 - p_1) \left( x^l - l + q S_2^{a_2 a_2} + (1 - q) S_2^{a_1 a_2} \right) \right] \\ = \frac{165}{1741} \left[ \frac{52}{100} \left( 1300 - h + 394 \right) + \frac{48}{100} \left( 500 - l + q 394 \right) \right] + \\ & \left( 1 - \frac{165}{1741} \right) \left[ \frac{1}{100} \left( 1300 - h + 394 \right) + \frac{99}{100} \left( 500 - l + q 394 + (1 - q) 414 \right) \right] \end{aligned}$$

Simplifying then substituting in the participation constraint, we observe that profits are increasing in h, so the optimal contract in the range maximizes h (and via participation, minimizes l), which gives  $h - l \rightarrow \frac{100}{3}$ . Combining this with the participation constraint gives  $h \rightarrow 518$ :

$$V_{B23} = \frac{1637\,702}{1741} - \frac{2539}{43\,525}h - \frac{40\,986}{43\,525}l = \frac{3349}{3482}h - \frac{76\,879}{1741} \rightarrow \frac{3349}{3482} * 518 - \frac{76\,879}{1741} \simeq 454$$

• In (B.4) and (B.5), expected profits when the period 1 contract satisfies participation with equality for  $a_2 \ (l = \frac{6275}{6} - \frac{13}{12}h)$  are (Note: p vanishes, so no need to substitute):

$$V_{B45} = \pi_B \left[ p_2 \left( x^h - h + p S_2^{a_2 a_2} \right) + (1 - p_2) \left( x^l - l \right) \right] + (1 - \pi_B) \left[ p_1 \left( x^h - h + p S_2^{a_2 a_2} + (1 - p) S_2^{a_1 a_2} \right) + (1 - p_1) \left( x^l - l + S_2^{a_1 a_2} \right) \right] \\ = \frac{5}{5127} \left[ \frac{52}{100} \left( 1300 - h + p394 \right) + \frac{48}{100} \left( 500 - l \right) \right] + \frac{5122}{5127} \left[ \frac{1}{100} \left( 1300 - h + p394 + (1 - p) 414 \right) + \frac{99}{100} \left( 500 - l + 414 \right) \right]$$

Simplifying then substituting in the participation constraint, we observe that profits are increasing in h, so the optimal contract in the range maximizes h (and via participation, minimizes l), which gives  $h - l \rightarrow \frac{40}{17}$ . The participation constraint then gives  $h \rightarrow \frac{42766}{85}$ :

$$V_{B45} = \frac{1575\,688}{1709} - \frac{897}{85\,450}h - \frac{84\,553}{85\,450}l = \frac{43\,537}{41\,016}h - \frac{2314\,547}{20\,508} \rightarrow \frac{43\,537}{41\,016}\frac{42\,766}{85} - \frac{2314\,547}{20\,508} \simeq 421$$

• In (B.6), the firm offers any contract with  $h - l < \frac{100}{51}$  that satisfies participation with equality,  $\frac{1}{100}h + \frac{99}{100}l - 1 = 500$ , as any such contract yields the same expected profits:

$$V_{B6} = \frac{1}{100} \left( 1300 - h + S_2^{a_1 a_2} \right) + \frac{99}{100} \left( 500 - l + S_2^{a_1 a_2} \right) = \frac{1}{100} 1300 + \frac{99}{100} 500 - 501 + 414 = 421.$$

(B.1) yields the highest expected profits, so the overall equilibrium results in burnout. Furthermore, the resulting expected profits are 460, while the first-best profits (from  $a_2, a_2$ ) are 808. Therefore, only  $\frac{460}{808} \approx 57\%$  of the first-best is achieved in equilibrium.

Even lower percentages are possible. However, the surplus,  $S^m$ , generated by the myopic first-best effort (then quitting in period 2) is greater than or equal to the period 1 surplus  $S_1^*$  generated by  $e_1^*$  and greater than the period 2 surplus  $S_2^*$  generated by  $e_2^*$  (as this is less than that generated by exerting  $e_2^*$  in the first period then quitting). Thus, we obtain

$$\frac{S^m}{S_1^* + S_2^*} \ge \frac{S^m}{S^m + S^m} = \frac{1}{2}.$$

## F Proof that the given strategies form an equilibrium in Example 2

The expected utility of the salesperson for each pair of actions (including q in period 2) are given in the following table, which establishes that  $(e_1^*, e_2^*) = (a_2, a_2)$  is indeed a best response to  $w_1 = (l_1, h_1) = (\frac{1454}{3}, 518)$  and  $(f_2, s_2) = (101, 901)$ .

$\mathrm{P1}\downarrow\ \mathrm{P2}\rightarrow$	$a_1$	$a_2$	$a_3$	q
$a_1$	592	999	998	484
$a_2$	607	1000	993	500
$a_3$	309	567	565	500

The firm also plays a best response, since it extracts all the surplus of the first-best pair of actions.